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ABSTRACT

This preliminary study examined the meanings, models, and strategies of rural students in grades 3-8 for solving simple, whole number division problems. Findings suggested that students have multiple meanings for division words such as "share" and "fair share." Cultural factors and diversity in the classroom may be associated with these multiple word meanings. Students in this study worked from three division models: partitive, quotative, and splitting. The majority of students selected the partitive model as their model of choice. Younger students selected the quotative model more than older students. Among four strategies, older students used division facts more often than younger students. Younger students depended on addition/subtraction strategies more than older students. A number of students at every grade level used multiple strategies. (Author)

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RURAL STUDENTS' INFORMAL KNOWLEDGE OF DIVISION

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This preliminary study examined the meanings, models, and strategies of rural students in grades 3-8 for solving simple, whole number division problems. Findings suggested that students have multiple meanings for division words such as *share* and *fair share*. Cultural factors and diversity in the classroom may be associated with these multiple word meanings. Students in this study worked from three division models: partitive, quotative, and splitting. The majority of students selected the partitive model as their model of choice. Younger students selected the quotative model more than older students. Among four strategies, older students used division facts more often than younger students. Younger students depended on addition/subtraction strategies more than older students. A number of students at every grade level used multiple strategies.

The division ideas of rural students in grades 3-8 were examined in this preliminary study. Previous studies investigated children's and teachers' division concepts and processes in relation to a number of external variables including problem type, context, number type, representations, and rule violations or misconceptions (Harel, Behr, Post, & Lesh, 1994; Greer, 1992; Tirosh & Graeber, 1990). Children in grades 1-3 were interviewed by Kouba (1989) to identify the division strategies used to solve simple division problems. Fischbein, Deri, Nello, and Marino (1985) concluded that the model of choice among students in grades 5 and 7 was the partitive division model. The quotative model influenced only grade 9 students' choices. Others have proposed that there are more division models, and of particular interest to us was Confrey's (1994) *splitting* model.

Division is one strand of the multiplicative conceptual field described by Vergnaud (1994). He asserted that the multiplicative conceptual field should first consider the intuitive, implicit, or informal mathematical knowledge of students. The theoretical framework of Vergnaud (1994) reflects the constructivism of Vygotsky who included a described teaching as the mediated meanings of situations, words, and symbols between teacher and student (Vygotsky, 1986). Teaching provides the social interactions, but from a constructivist perspective the learning is a personal process, unique to every student. Vygotsky (1986) observed that children before adolescence have very different meanings than those of adults for situations, words, and symbols (Voight, 1994). To better understand the multiple meanings, models, and strategies that students bring to the classroom, this study investigated the informal knowledge used by rural children in grades 3-8 to solve simple, whole number division problems.

Method

The students in this preliminary study were from a large, rural county in Southeastern United States. Within the county there was a wide range of cultures. Many students' families have lived in their farming communities for generations, other families are transient or migrant agricultural workers. In this county school system

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there are 16,067 K-12 students of which 72% are white and 23% are African American. Hispanics (3%) make up a small percent of the school population followed by Native Americans and Asian students who together account for less than 1% of the student population. In March 1995, 35% of the K-12 school population received free or reduced lunch indicating low socio-economic status.

Data for this study were collected in two ways. Teachers of grades 3-8 conducted structured video interviews of some of their students (n=55) and gave a paper/pencil assessment to all of their students (n=451). In the structured interviews, students were encouraged to talk about *sharing* in their lives. Then teachers asked students to determine if and explain why a particular distribution of candy was a *fair share*. The interview also included a simple, open-ended division problem that suggested *sharing division* and involved dividing a bag of candy. The first item on the paper/pencil assessment asked students to explain in words and pictures how they would solve a simple, partitive division problem. Next students were asked to write a story problem from a picture that suggested partitive division.

Analyses and Results

Data from the interviews and the paper/pencil instrument were examined to determine the meanings that 3-8 students gave for *share* and *fair share*, their models for division, and the strategies that they used to solve simple, partitive division problems. The analyses of the data involved multiple sorts to code the responses. Agreement between the categories established by previous studies and our categories provided information for final adjustments to the categories. Then the data were tabulated to reflect the frequency of each category. Student profiles were used to report the frequency of strategy responses because students had more than one strategy for a single item.

Meanings

Sharing is recommended in methods textbooks as a meaningful way to introduce young children to partitive division. However, Vygotsky (1986) stated that children, up until adolescence, have very different meanings of words than adults, although they have learned how to use the words to communicate with adults. In the interviews, students were asked about their definitions of *sharing*, and to explain if the teacher's arbitrary distribution of candy was an example of *fair share*. Multiple sorts of these responses were coded to establish the following categories of meanings for both *share* and *fair share*: 1) equal amounts in meaning, 2) mathematical meaning, and 3) cultural meaning. The first category was tabulated if students mentioned equal shares. The second category was tabulated if students gave a description of division. If the student's definitions of *share* and *fair share* indicated that cultural factors were involved, then tabulation was made in the third category.

Among 55 interviewed students, there were 15 students whose definitions of *sharing* included ideas of equality, and 31 students whose definitions of *fair sharing* included ideas of equality. Older students tended to include ideas of equality more than younger students, for the meanings of both words. The analysis of whether students associated division with *sharing* and *fair share* gave the following results. Only 4 students included division in their definitions of *sharing* (1 for 3-4 gr.; 2 for 5-6 gr.; 1 for 7-8 gr.). Two students included division as part of their definition of *fair share*. Many students, who gave non-mathematical definitions, showed great diversity in their meanings of *share*, and indicated a wide variety of cultural values. They included:

- To loan personal items such as toys, clothes, etc.
- Taking turns with another person.
- Doing things with another person.
- To keep a secret with a friend or sibling.
- If they have nothing, you give them something of yours.
- Giving to someone if they deserve it.
- To give others the same amount, including one's self so nobody gets upset.
- If you have something that the other person wants, and they have something you want, then you give each other those things.
- To give the other person more because I can get whatever I want anytime.
- Everyone is treated fairly when you have something.
- Older people, like parents or older siblings, can get more than younger children.
- Not sharing all the candy so that you can save some for another day. Only take a few.

Among students in grades 3-4, 71% included cultural factors in their definitions of *sharing*. For students in grades 5-6 and grades 7-8, 74% and 59% respectively included cultural factors in their definitions. When asked if a particular distribution was a *fair share*, the evidence of cultural values decreased greatly among all students' meanings (5% overall) and as previously seen, a corresponding increase in notions of equality among groups was noted.

Models

Primitive models used in solving division problems were reported by Fischbein et al. (1985) and included the partitive and the quotative models of division. Fischbein described the partitive model as *sharing division*, where something or a

collection of things is divided into a number of equal parts or groups. He defined the quotative model as *measurement division*, where the student is required to find how many times a given quantity is contained in a larger quantity. In addition, Confrey (1994) has proposed a third primitive model of *splitting division*. Much like a binary tree, the students make successive halvings or pairings in the splitting process to produce multiple versions of the original. These new versions are created by either magnifying or shrinking the original version. Interview transcripts in our study were examined to tabulate how many students used each of the three models. Also, note was made of the students who determined the dividend before solving the open-ended problem.

Table 1 presents the frequency distributions of the different models in percent for grades 3-4, 5-6, and 7-8. We found that more than half of the students (51%) did not count the candy to determine the dividend. Sixty-four percent of the students in grades 3-4, 56% of the students in grade 5-6, and 39% of the students in grades 7-8 did not know the dividend before starting their solutions.

Table 1
Frequency of Primitive Models Used by Students in Grades 3-4, 5-6, and 7-8 to Solve an Open-Ended Division Problem

Model	% Categorical Responses (n) 3rd & 4th grade (n=14)		5th & 6th grade (n=18)		7th & 8th grade (n=23)		totals (n=55)	
Partitive	50	(7)	67	(12)	78	(18)	67	(37)
Quotative	36	(5)	28	(5)	22	(5)	27	(15)
Splitting	14	(2)	6	(1)	–		5	(3)

The partitive model for division was used more frequently by students at all grade levels. This may be because the problem involved dividing candy, suggesting a sharing model of division. However, 27% of the students selected a quotative model for the problem. Few students (5%) used the splitting model. A larger percent of the older students used the partitive model while the younger students used it less. Among those selecting a quotative model, there was a larger percent of younger students and a smaller percent of older students who selected the model. No students in grades 7-8 selected the splitting model.

Strategies

The paper/pencil instrument was used to collect data about the division strategies of students in grades 3-8. Multiple sorts established the categories of the solution strategies of simple, partitive division problems. These categories were compared to those reported by Kouba (1989) and modified as follows: 1) addition/subtraction, 2) dealing out, 3) multiplication, and 4) division. Two dealing out strategies were noted among the students – one in which the student distributed one or two to each group (divisor) consecutively, and the other in which the stu-

dent dealt out all the objects (quotient) at one time to a number of groups. The first dealing out process required multiple circuits of the groups up to the limit of the dividend. The second dealing out process required only one circuit of dealing out where the number of groups formed were limited by the dividend. Since students may have used more than one strategy within a response, student profiles were used to represent the different combinations.

The profiles of students' division strategies are reported below in Table 2. In the student profile, a "1" indicated that a particular strategy was present, while a "0" indicated the absence of the strategy. Reading from left to right the cell combinations are addition/subtraction, dealing out, multiplication, and division. For our study, a "1001" profile indicated the student employed an addition/subtraction strategy and a division strategy to solve simple, partitive division problems. Also, the profile indicates that the student did not use any dealing out or multiplication strategies. Of the 451 students who took the paper/pencil instrument, there were 134 who gave incomplete or no response. The percents shown in Table 2 are calculated on the 317 students whose answers indicated a particular strategy.

Table 2

Student Profiles for Grades 3-4, 5-6, and 7-8 and the Frequency of Strategies Used to Solve Partitive Division Word Problems

Profile	% Categorical Responses (n)							
	3rd & 4th grade (n=64)		5th & 6th grade (n=102)		7th & 8th grade (n=151)		totals (n=317)	
0001	22	(14)	47	(47)	56	(84)	46	(145)
0010	23	15)	25	(25)	18	(27)	21	(67)
0100	8	(5)	5	(5)	3	(4)	4	(14)
1000	31	(20)	6	(6)	5	(8)	11	(34)
0011	5	(3)	8	(8)	9	(14)	8	(25)
0101	–	1	(1)	3	(4)	1	(5)	
0110	–	1	(1)	–	< 1	(1)		
1001	6	(4)	4	(4)	4	(6)	4	(14)
1010	5	(3)	1	(1)	1	(2)	2	(6)
1100	–	2	(2)	–	1	(2)		
1011	–	2	(2)	–	1	(2)		
0111	–	–	1	(2)	1	(2)		

Generally, older students selected division as a strategy more than younger students. Younger student preferred addition/subtraction strategies more than older students. There were a total of 55 students who had more than one strategy to explain their solutions to the partitive division word problems.

Discussion and Summary

This was a preliminary study to examine the informal meanings, models, and strategies of rural students in grades 3 - 8 when solving primitive division problems. Primitive is a descriptor used in the sense that both the quotient and the

divisor were whole numbers greater than one but smaller than the whole number dividend (Harel, et al., 1994; Fischbein et al., 1985). It appears, that for some rural students there were strong cultural values associated with and affecting their meanings of *sharing*. Giving someone more because one has enough may be indicative of the value that farmers place on helping one's neighbors. Saving some candy for another day seemed to us to be a survival strategy for a child living in poverty. The multiple meanings of *share* and *fair share* suggested that teachers cannot assume that all of their students have the same meanings of words used in the mathematics classrooms, especially if the students are culturally diverse. We view the different meanings of words of teachers and students as an interesting research area to pursue. Our findings concerning models are somewhat different than Fischbein et al. (1985) with respect to older students' success with quotative division. This may be because Fischbein's quotative problems involved decimal numbers while ours were whole numbers. We found that a higher percent of the younger students selected the quotative model when compared to the percent of older students. A future research question for investigation, as suggested by Kouba (1989), is to study individual students' use of multiple models of division, and in particular, the splitting model. We found that students used multiple strategies to solve these division problems, and perhaps they use multiple models as well. The extent to which students can apply the strategies identified in this study to division problems involving other models and division of rational numbers are other areas for future research.

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